Multi-material interface reconstruction from the moment data

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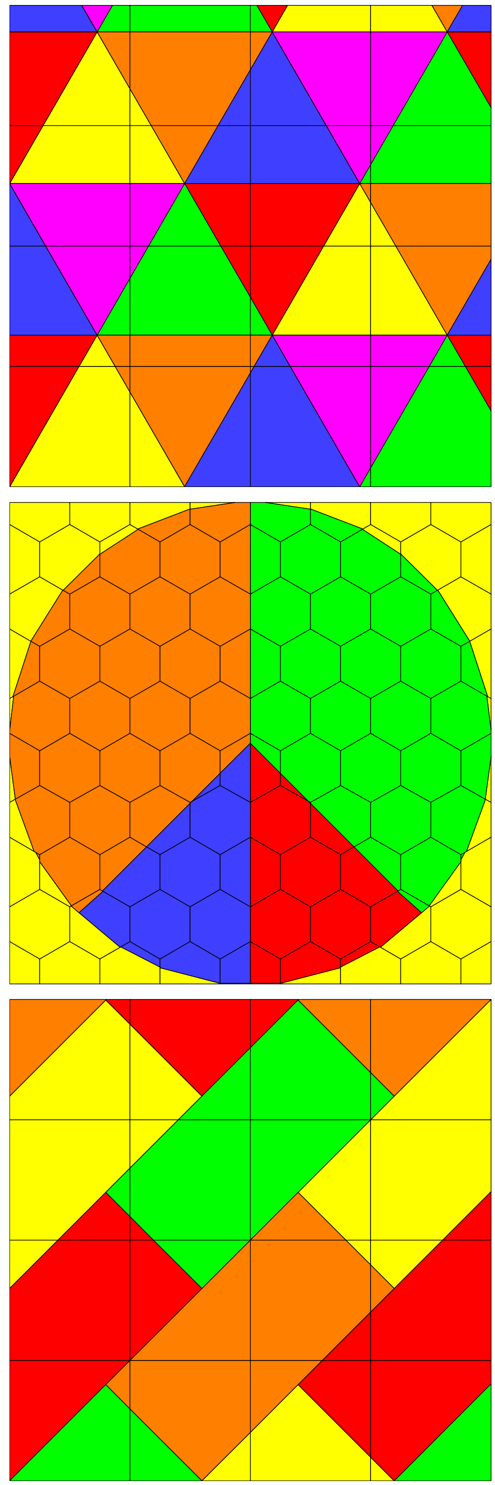
The purpose of the Moment-of-Fluid (MoF) technique is reconstruction of the mixed-cell material interfaces from the moment data: the volumes and centroids of materials. A special two-material case of MoF reconstruction has already been presented in [2]. MoF algorithm defines location of the linear interface in a two-material mixed cell *by minimizing the defect of the first moment over all the volume-preserving cell partitions.* The same governing principle can be used to perform interface reconstruction in a multi-material mixed cell.

The mixed-cell partitioning can be a real problem in case of multiple materials. We demonstrate how the two-material MoF interface reconstruction algorithm can be used to perform a polygonal partitioning of a mixed cell with $M \geqslant 3$ materials. Basically, we follow the strategy of the multi-material Volume-of-Fluid (VoF) method and use the two-material interface reconstruction algorithm for extracting materials from the mixture one by one. There is an essential difference though: the MoF interface reconstruction does not require the user to specify the material order explicitly. *The right order is determined automatically* by trying all $M!$ possible material orders and finding the one that results in the minimal defect of the first moment.

Such an algorithm has combinatorial complexity in the number of materials: to get the answer, one has to try all $M!$ material orders. On the other hand, it is reasonable to expect only a limited number of the mixed cells in the whole computational grid to contain 3+ materials. Therefore, for a moderate M, the computational overhead, associated with the optimal order search, is not likely to be significant. Also, various material orders



Examples of the multi-material MoF reconstruction.

can be effectively tried in parallel.

The search of the best mixed-cell partition does not limit the choice of partitioning scheme in any way. Therefore, in order to achieve a lower defect of the first moment, one can expand the family of trial partitions at will. For instance, instead of extracting materials from the mixture in series, one can separate them according to the "divide-and-conquer" principle: choose an arbitrary $m < M$, separate the mixture of materials $\overline{1,m}$ from $\overline{m+1,M}$, and then recursively apply this algorithm to each submixture containing *2+* materials. This procedure allows to generate $M!(M\text{-}1)!$ trial *B-tree partitions* to choose from, which significantly increases the chances of finding an approximate partition that fits given moment data best. As a result, the Multi-Material MoF algorithm can reconstruct an arbitrary C^2-differentiable B-tree partition with 2nd-order accuracy, which can hardly be achieved in the VoF context.

Although we explicitly address 2D case only, it is clear that all the partitioning and ordering strategies described are dimension-independent and are applicable in 3D.

Compared to the competitors, the VoF interface reconstruction techniques, MoF algorithm shows higher accuracy, allows uniform processing of internal and boundary cells, and is truly robust in treating multi-material mixed-cells.

References

[1] V. Dyadechko and M. Shashkov. Multi-material interface reconstruction from the moment data. Technical Report LA-UR-06-5846, Los Alamos National Laboratory, Los Alamos, NM, Aug 2006.

[2] V. Dyadechko and M. Shashkov. Moment-of-Fluid interface reconstruction. Technical Report LA-UR-05-7571, Los Alamos National Laboratory, Los Alamos, NM, Oct 2005.

To get more information on MoF technique go online at
`http://math.lanl.gov/~vdyadechko/research`
or contact Vadim Dyadechko.

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